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Photon tunneling and light coupling to planar slab waveguides

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Abstract : We show that the guided modes from one slab waveguide in the optical integrated circuits (OIC) can be coupled and diffused to another slab waveguides due to photon tunneling while it is forbidden in normal classical treatment. Using this idea, we can estimate and evaluate the crosstalk level in another optical channel and design the suitable and optimal distance between two waveguides for small and determinable coupling rate. The Helmhoetz wave equation has been used to calculate the crosstalk level and transit time during coupling.

Keywords : Light coupling, photon tunneling, crosstalk level, optical intergrated circuits.

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1. Introduction

One of the most interesting research areas in photonic engineering is integrated optics or optical integrated ciucuits (OIC). According to electronic integrated circuits, in which we consider the classical treatment for understanding the behaviour of the system in this domain, the classical Maxwell equations describe the transfer functions in relation to these systems. As the dimensions decreases, the quantum mechanical effects can be observed. One of the most interesting and critical phenomena which can be seen in this domain, is the minimum distance between the slab or channel waveguides in OIC. For determining this minimum distance, we must consider the coupling of one waveguide to adjacent waveguides. Our aim in this paper, is to discuss about photon coupling and diffusion. In recent years, the tunneling time of a particle passing through a potential barrier (opaque classically), has been one of the most striking subjects. Most of these works are based on numerical and analytical analysis of Schrödinger equation with some initial conditions on the wave packets [1–3], and have been proved experimentally [4–7]. The tunneling time of a particle can be obtained in different ways [8–21]. The analogy between quantum tunneling of particle and evanescent electromagnetic wave found in a low-dielectric constant region (separating two regions of high dielectric constants), can be considered and one can find a suitable approach for calculation of the tunneling time of photon from this classically forbidden region. We refer to Figure (1-b) and consider a light beam impinging from a dielectric medium with index n_1 onto a dielectric slab waveguide with index $n_2 < n_1$. If the incident angle is greater than the critical value $\theta_c = \sin^{-1}(n_2/n_1)$, all of the light beam must be reflected, but in practice, some part of

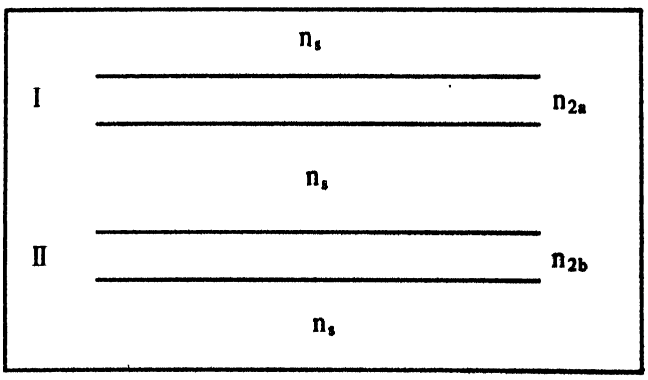


Figure 1a. To view of waveguides in substrate.

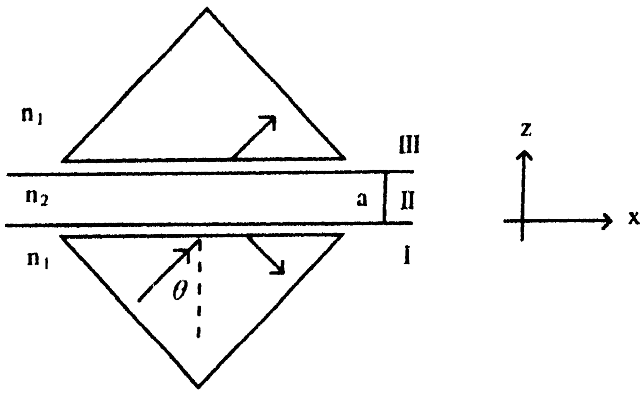


Figure 1b. Structure for light coupling.

it tunnels through the slab and emerges in the second dielectric medium with index n_1 and another part of this light beam reflected. In high precision measurement systems and in OIC with high degree of integration, the photon tunneling and diffusion have two main disadvantages as :

(i) Increasing the crosstalk level (decreasing the sensitivity).

(ii) Increasing the minimum optical layer thickness (decreasing the integration density).

In this work, we propose the analytical relations for investigation of these effects. For this purpose, the Helmholtz wave equation has been used. Therefore, the concept of potential energy in the particle tunneling changes to the index of refraction in the photon tunneling and evanescent electromagnetic wave packet propagation [8–10]. In this paper, we assume for simplicity, the medium to be lossless, non-dispersive and linear. We consider an analytical solution to the Helmholtz equation to derive an expression for the tunneling time and other important quantities. It has been shown that the tunneling time depends on refractive index, the width of barrier, incident angle, and the frequency of incident light upon the barrier. Using the tunneling time and transmission coefficients, we can obtain the crosstalk strength and minimum distance between two adjacent waveguides in the optical integrated cirucuits. Organization of this paper is as follows :

In Section 2, the basic idea for tunneling time and transmission coefficient in discussed. The dependency of light coupling and crosstalk level to system parameters in OIC, presented in Section 3, followed by conclusion in Section 4.

2. Photon tunneling

We study an one-dimensional problem in which wave packet propagates along one direction, say z. Let us consider a wave packet moving along z-axis incident on a region $[0, a]$ with refractive index $n(z)$, as depicted in Figure 2. For simplicity, the barrier can be approximated as a square one, in which $n(z)$ is constant in regions I, II and III but different from one region to another. The frequency of the incident wave is such that the region II is classically forbidden. We also, for simplicity, assume that $n(z)$ is identical in regions I and III. We can take the components

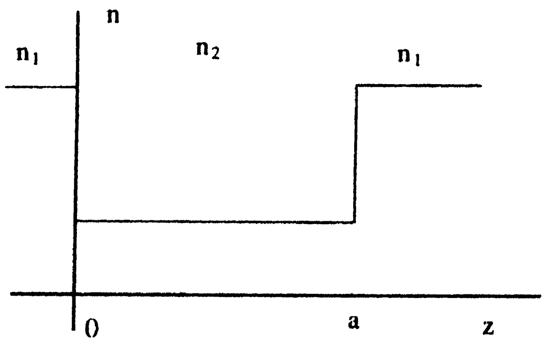


Figure 2. Index of refraction barrier

of the electric and magnetic fields as E and B respectively. In general, we consider that the propagation of wave through the barrier can be described by a scalar field Ψ representing the electric field in this case and the Schrödinger wave function in particle case. In order to obtain the tunneling time by particle, one can solve the Schördinger equation of motion. But in our case, we must consider the Helmholtz equation [11,12, 14–18] for the transverse electric field mode (TE) such as :

$$\nabla^2 \Psi + K^2 \Psi = 0. \tag{1}$$

This equation has the following solutions in regions I, II and III respectively.

$$\Psi_{\mathrm{I}} = e^{ik_0 z} + Re^{-ik_0 z}, \tag{2}$$

$$\Psi_{\mathrm{II}} = Ae^{-\chi z} + Be^{\chi z}, \tag{3}$$

$$\Psi_{\mathrm{III}} = Te^{ik_0(z-a)}, \tag{4}$$

where $k_0 = \frac{\omega}{c} n_0$ and $k_0 = \frac{\omega}{c} n = i\chi$ are the wave vectors in regions I (or III) and II respectively.

Note that we have ignored the time dependent factor $e^{i\omega t}$. As is well known, the coefficients R, T, A and B can be calculated by considering boundary conditions. But in this case, we are interested to find T wihich is given as :

$$T = \frac{-4i\chi k_0}{(\chi - ik_0)} \cdot \frac{e^{-\chi a}}{1 - e^{-2\chi a} r^2}, \tag{5}$$

where $r = \left(\frac{\chi + ik_0}{\chi - ik_0}\right)$. By substituting $-ik = \chi$, this equation could be separated to real and imaginary parts, so that the phase difference is given by :

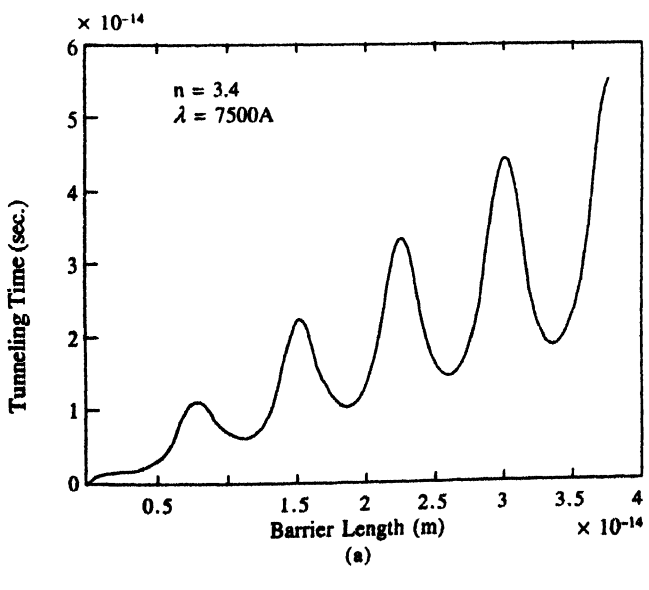
$$\phi = \tan^{-1}\left(\frac{\sin ka(1 - r^2\cos 2ka) + r^2 \sin 2ka.\cos ka}{\cos ka(1 - r^2 \cos 2ka) - r^2 \sin ka.\sin 2ka}\right). \tag{6}$$

There are very different way to calculate the tunneling times [3,9,13,16,17]. Here we consider the transit time τ for a wave packet propagating through a given region which can be measured as the interval between the arrival times of the signal envelope at the two ends of that region whose distance is a. In general, if the wave packet has a group velocity v_g, $\tau = a/v_g$ [18].

Since $v_g = d\omega/dk$ (k wave vector and ω angular frequency), we can write

$$\tau = d\phi/d\omega, \tag{7}$$

where $d\phi = adk$ is the phase difference of wave packet in that region. Now, by using equations (6), (7) and $k = \frac{\omega}{c}n$ and $r^2 = \left(\frac{n - n_0}{n + n_0}\right)$, the transit time can be obtained as



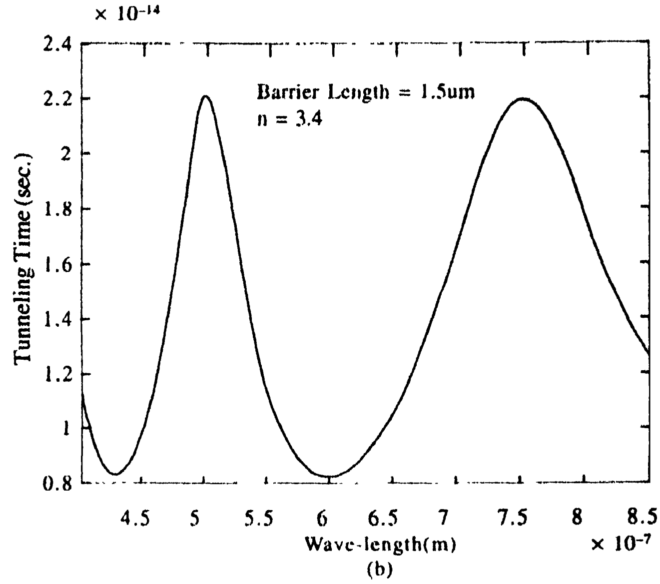


Figure 3. Tunneling time *vs* barrier length and wave-length in macroscopic scale; (a) tunneling time-barrier length, (b) tunneling time-wave-length.

$$\tau = \frac{d\phi}{d\omega} = \frac{\frac{na}{c}\left(1 - r^4\right)}{1 + r^4 - 2r^2 \cos 2ka} \tag{8}$$

where c is the speed of light free space and n is the refractive index of region II and a is the barrier width. Figure 3 shows the variation of tunneling time *versus* barrier length and wavelength.

3. Crosstalk in OIC and photon tunneling

According to Figure (1-b), we have two-dimensional problem and is treated as follows. The wave vectors k_1, k_2 in regions I (or III) and II satisfy the following equations :

$$k_1^2 = k_x^2 + k_0^2,\ k_2^2 = k_x^2 - \chi^2, \tag{9}$$

where k_x is the x component of k_1 and k_2 and also k_0, χ are defined in Section 2. According to our results obtained in Section 2, the dispersion relations in regions I (or III) and II are respectively given as :

$$k_1 = \frac{\omega}{c}n_1,\ k_2 = \frac{\omega}{c}n_2 \ \text{or}\ v_1 = \frac{\omega}{k_1},\ v_2 = \frac{\omega}{k_2}$$

Now, to determine the dependence of the tunneling time on incident angle we propose an equivalent relation for tunneling time by use of basic definition given in eq. (7). To this end, we rearrange the transmission coefficient for $\chi a \gg 1$ as

$$T = \frac{-4i\chi k_0 e^{-\chi a}}{(\chi - ik_0)^2}$$

$$= \frac{2e^{-\chi a}}{1 - i\dfrac{k_0{}^2 - \chi^2}{2k_0\chi}}. \tag{10}$$

Using definition developed in Section 2, we obtain the phase difference of transmission time as

$$\phi = \tan^{-1}\left[\frac{\left(\dfrac{k_0}{\chi}\right)^2 - 1}{2\left(\dfrac{k_0}{\chi}\right)}\right]. \tag{11}$$

So, the tunneling time of photon can be formulated as

$$\tau = \frac{d\phi}{d\omega} = \frac{2}{1 + \left(\dfrac{k_0}{\chi}\right)^2}\frac{d}{d\omega}\left(\frac{k_0}{\chi}\right) \tag{12}$$

which can be simplified as

$$\tau = \frac{2}{\chi k_0}\left[\frac{\chi^2}{\chi^2 + k_0^2}\left(k_0\frac{dk_0}{d\omega}\right) + \frac{k_0^2}{\chi^2 + k_0^2}\left(-\chi\frac{d\chi}{d\omega}\right)\right]. \tag{13}$$

Also, we can use the following relations for more simplification of eq. (13).

$$k_0\frac{dk_0}{d\omega} = \frac{k_1}{v_1},$$

$$-\chi\frac{d\chi}{d\omega} = \frac{k_2}{v_2}. \tag{14}$$

Finally, using eqs. (9, 14) and some simple mathematical manipulations, we obtain the final exact relation for tunneling time as

$$\tau = \frac{1}{\omega}\frac{2k_x^2}{\chi k_0}, \tag{15}$$

where using Figure (1-b), $K_x = k_1 \sin\theta = \frac{\omega}{c} n_1 \sin\theta$, $k_0 = k_1\cos\theta = \frac{\omega}{c} n_1 \cos\theta$ and $\chi = (k_1^2 \sin^2\theta - k_2^2)^{1/2}$. We finally get

$$\tau = (1/\upsilon)(n_1 \sin^2\theta / \pi\cos\theta\ (n_1^2 \sin^2\theta - n_2^2)^{1/2}). \tag{16}$$

Figure 4 shows the variation of the tunneling time *versus* incident angle and incident light frequency. The transmission coefficient as function of index of refraction, barrier length and incident angle are shown in Figure (5–7). Also, we can calculate the optimum barrier length, index of refraction and other parameters for minimizing the crosstalk level and tunneling rate. For this purpose, the transmission coefficient given by eq. (5),

$T = \dfrac{-4i\chi k_0}{(\chi - ik_0)^2}\dfrac{e^{-\chi a}}{1 - e^{-2\chi a}r^2}$ and $r = \dfrac{\chi + ik_0}{\chi - ik_0}$, can be used.

So, as a acceptable rule in the engineering and science, the $|T| = e^{-1}$ can be used as a criteria for considerable penetration and tunneling quantity. So, the minimum barrier length for a given index of refraction barrier pattern, can be obtained by inserting $|T| = e^{-1}$ eq. (5) as

$$a_{\min} = \frac{1}{\chi}\ln\left[e\frac{4n_2n_1}{(n_2 + n_1)^2}\right] \quad \text{for } \chi a >> 1. \tag{17}$$

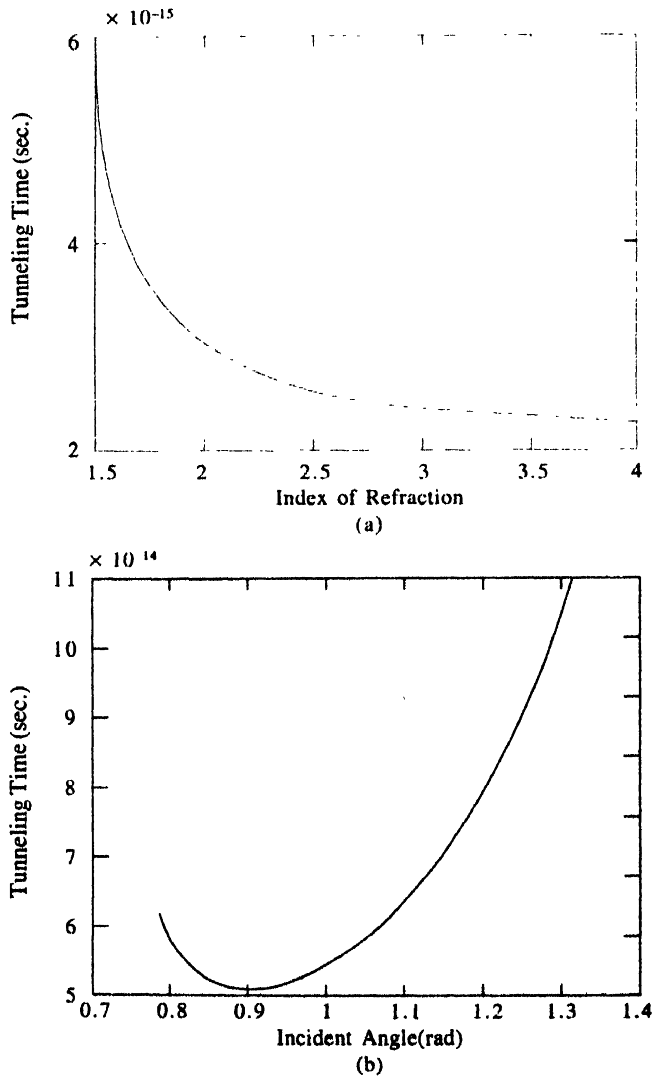


Figure 4. Tunneling time *vs* index of refraction and incident angle : (a) tunneling time-index of refraction, (b) tunneling time incident angle (n_1 = 1, n_2 = 1.5, $\omega = 2\pi 1.5 \times 10^{14}$).

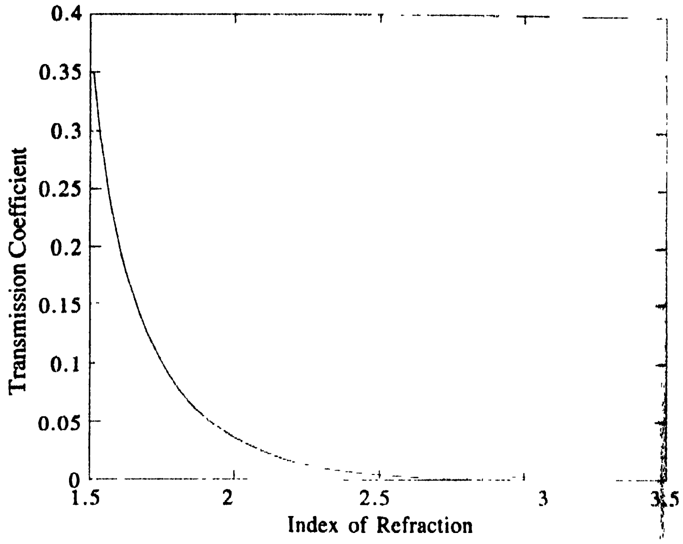


Figure 5. Transmission coefficient *vs* index orefraction

$\left(n_1 = 1,\ n_2 = 1.5,\ \omega = 2\pi 1.5\times10^{14},\right.$

$\left. a = 1.5\times10^{-16},\ \theta = \sin^{-1}\left(\frac{1}{1.5}\right)\right)$

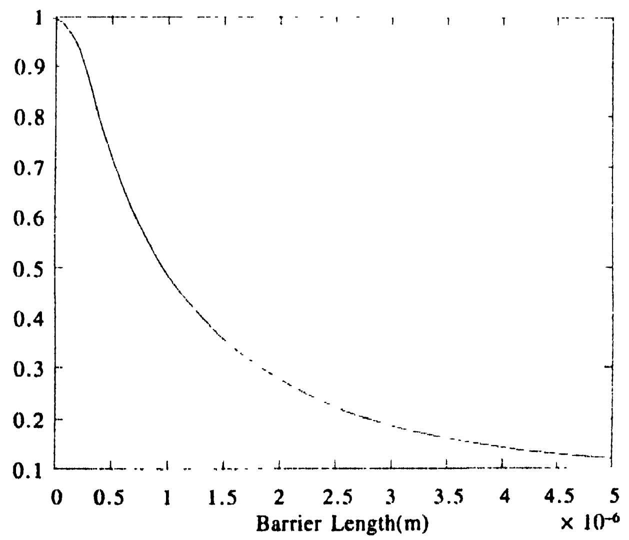


Figure 6. Transmission coefficient *vs* barrier length

$\left(n_1 = 1,\ n_2 = 1.5,\ \omega = 2\pi 1.5\times10^{14},\right.$

$\left. a = 1.5\times10^{-16},\ \theta = \sin^{-1}\left(\frac{1}{1.5}\right)\right)$

Figure 8 shows the minimum barrier length as the function of index of refraction and the frequency. In general case for *a*, by use of transmission coefficient, one can estimate the crosstalk strength level in second wave-guide as

$$\frac{I_{\text{noise}}}{I_{\text{noise}}} = |T|^2 \tag{18}$$

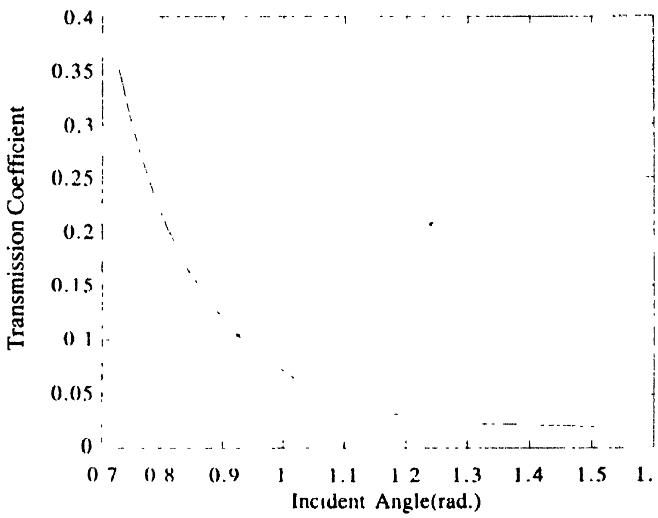


Figure 7. Transmission coefficient *vs* incident angle ($n_1 = 1$, $n_2 = 1.5$, $\omega = 2\pi 1.5 \times 10^{14}$, $a = 1.5 \times 10^{-16}$).

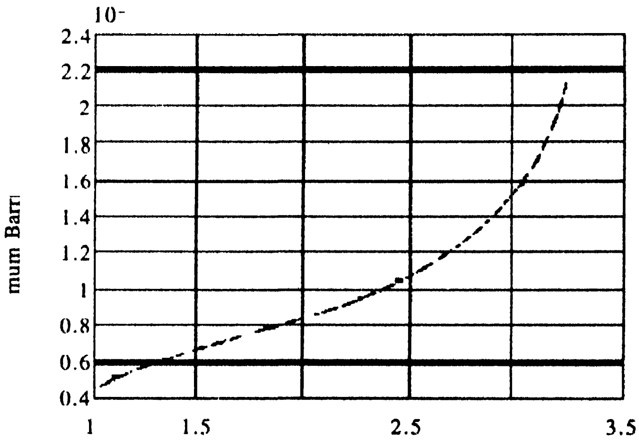


Figure 8. Minimum barrier length as Index of refraction

$n_1 = 3.5,\ \omega = 2\pi 1.5\times10^{14},$

$a = 1.5\times10^{-16},\ \theta = \sin^{-1}\left(\frac{1}{1.5}\right.$

4. Conclusion

In this paper, the light coupling into classically forbidden region is discussed using photon-tunneling concept for index of refraction barrier. We derive an exact relation between transmission coefficient and tunneling time for index of refraction barrier. In fact this barrier can not act really as an ideal reflector. Therfor, it is neccessary to calculate the crosstalk term due to tunneling which can be seen in Figures (3-8). In these figures, we demonstrate the dependence of transmission coefficient and tunneling time on various system parameters.

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